Key

Examples Day 2:

1. A box is made by cutting equal squares out of the corners of a rectangular piece of cardboard 6" by 16" and folding up the flaps. Find the dimensions of the box which will have maximum volume.

2. Determine the radius and height of the cylinder of maximum volume that can be obtained by revolving a rectangle of perimeter 24" about one of its sides.

Don't skip

On the graph $y = x^3$ take a point $P(x, y)$ subject to the condition $0 \le x \le 6$. Join point P to the point (6, 0) by a straight line and drop a perpendicular from P to the x-axis. These two lines and the x-axis form a right triangle. Find the value of x for which the area of the triangle is a maximum.

No skip

A grower estimates that if the crop of oranges is harvested now, the average yield of 80 pounds per tree can be sold at \$.40 per pound. From past experience, the owner expects the crop yield to increase at a rate of 10 pounds per week per tree, and the price to decrease at a rate of \$.02 per pound per week. When should the oranges be picked in order to attain maximum sale?

Homework Day 2:

1. The cube of a non-negative number is subtracted from the square of the same number. Find the number, which gives the greatest difference.

Challenge

2. The sum of two positive numbers is 20. If P is the product of one number and the square of the other, find the two numbers that will maximize P.

3. Find two positive numbers whose product is 64 and whose sum is as small as possible.

4. Find the number which when added to twice the square of the number will give the least sum.

5. Find two numbers whose difference is 22 and whose product is a minimum.

6. Three times the square of a number is subtracted from the cube of the number. If the number is non-negative, find the number that gives the least difference.

7. Cutting equal squares out of the corners of a piece of cardboard 12" by 12" and folding up the flaps makes a box. Find the dimensions of the box which will have maximum volume.

HW ANSWERS: 1. 2/3 2. 40/3 3. 8, 8 4. -1/4 5. -11, 11 6. 2 7. 2" by 8" by 8"

U7 L1 I6 Day 2 ANSWERS

M4

Day 2 Ex

W L H

1.) $V=(16-2x)(6-2x)x$

$=(96-32x-12x+4x^2)x$

$=96x-44x^2+4x^3$

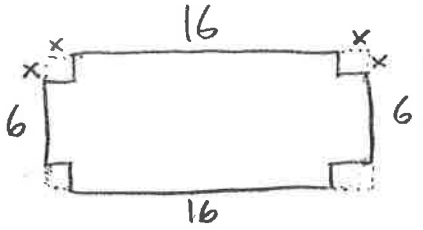
$\frac{dV}{dx}=96-88x+12x^2$

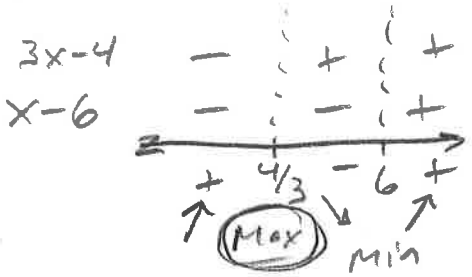
$=12x^2-88x+96$

$=4(3x^2-22x+24)$

$=4(3x-4)(x-6)$

$x=\frac{4}{3}$ $x=6$





$W=16-2(\frac{4}{3})=13.\overline{3}$ in

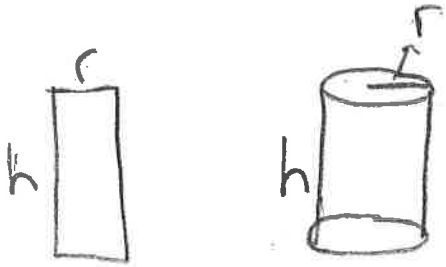
$L=6-2(\frac{4}{3})=3.\overline{3}$ in

2.) $2h+2r=24$ $V=\pi r^2h$

$2h=24-2r$

$h=12-r$

$V=\pi r^2(12-r)=\pi\cdot 12r^2-\pi r^3$

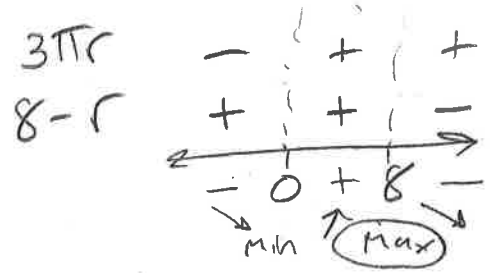


$\frac{dV}{dr}=24\pi r-3\pi r^2$

$0=3\pi r(8-r)$

~~$r=0$~~ $r=8$

Doesn't make sense



Radius = 8 in
Height = 4 in.

$24=2h+2(8)$

$8=2h$

$4=h$

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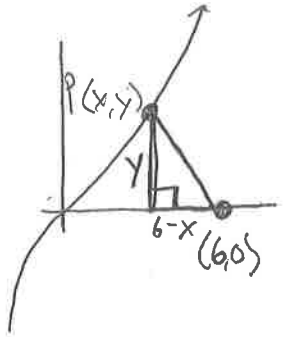
Day 2 EX Cont'd.

Max Area

3.) $A = \frac{1}{2}(6-x)y \qquad y = x^3$



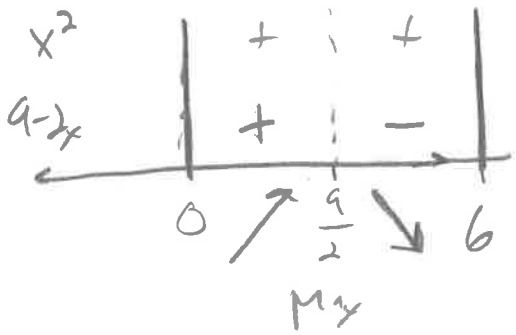
$A = \frac{1}{2}(6-x)x^3$

$A = 3x^3 - \frac{1}{2}x^4$

$A' = 9x^2 - 2x^3$

$A' = x^2(9-2x)$

$x = 0, \quad x = \frac{9}{2}$



$\boxed{x = \frac{9}{2}}$

Max Sale

4.) ~~SKIP!~~ S = sale, w = week number

Sale = yield · price

yield = $80 + 10w$

price = $40 - 2w$

$S = (80+10w)(40-2w)$

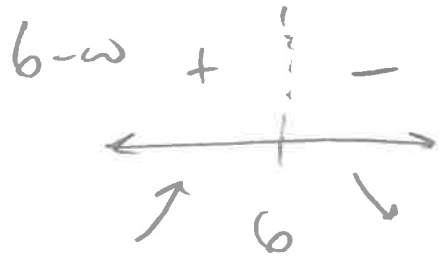
$S = 3200 - 160w + 400w - 20w^2$

$S = 3200 + 240w - 20w^2$

$S' = 240 - 40w$

$S' = 40(6-w)$

$w = 6$



$\boxed{\text{After 6 weeks}}$

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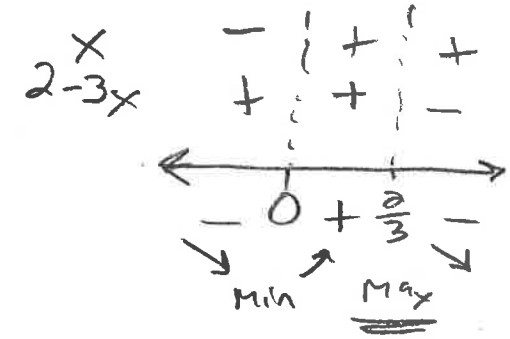
Day 2 HW 1.) $D = x^2 - x^3$ Max diff = ?

$D' = \frac{dD}{dx} = 2x - 3x^2$

$0 = 2x - 3x^2$

$0 = x(2 - 3x)$

$x = 0$ $\quad x = \frac{2}{3}$



★ 2.) $x + y = 20$ Max P

$P = x \cdot y^2$ $\quad x = 20 - y$

$P = (20 - y)y^2$

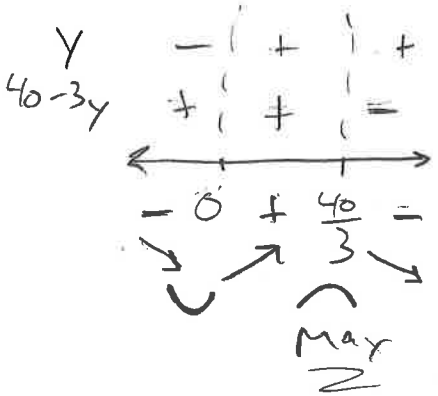
$P = 20y^2 - y^3$

$\frac{dP}{dy} = 40y - 3y^2$

$0 = y(40 - 3y)$

~~$y = 0$~~ Not positive $\quad y = \frac{40}{3}$

$x = 20 - 40/3 = \frac{20}{3}$



★ 3.) $xy = 64 \rightarrow y = \frac{64}{x}$ Min Sum = ?

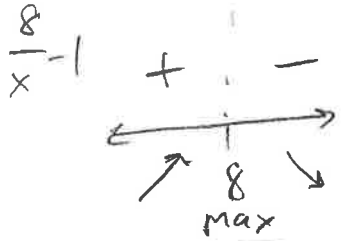
$S = x + y$

$S = x + \frac{64}{x} = x + 64x^{-1}$

$\frac{dS}{dx} = 1 - 64x^{-2} = -\left(\frac{64}{x^2} - 1\right) = \left(\frac{8}{x} + 1\right)\left(\frac{8}{x} - 1\right)$

$0 = \left(\frac{8}{x} + 1\right)\left(\frac{8}{x} - 1\right)$

~~$x = -8$~~ Not positive $\quad x = 8$



$x = 8$

$y = 8$

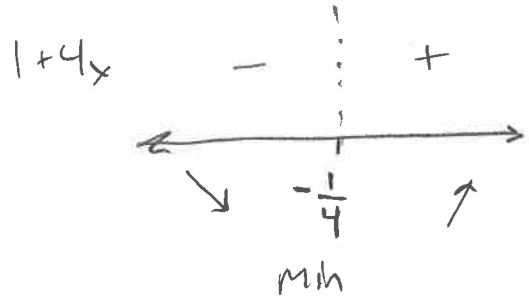
→

4.) $S = x + 2x^2$ Min sum = ?

$\frac{dS}{dx} = 1 + 4x$

$0 = 1 + 4x$

$\boxed{x = -\frac{1}{4}}$



5.) $P = xy$ Min Product = ?

$x - y = 22$

$x = 22 + y$

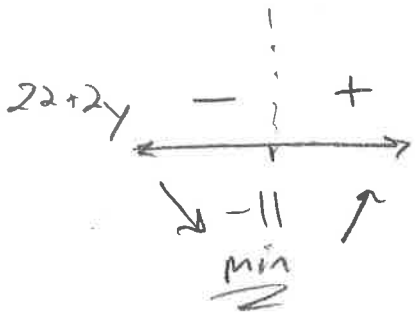
$P = (22+y)y = 22y + y^2$

$\frac{dP}{dy} = 22 + 2y$

$0 = 22 + 2y$

$y = \boxed{-11}$

$x = 22 + (-11) = \boxed{11}$



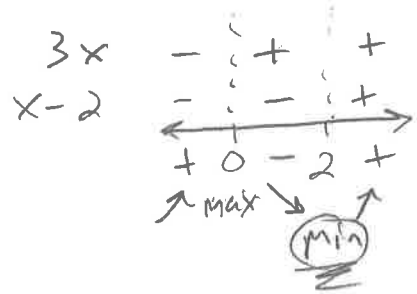
6.) $D = x^3 - 3x^2$ Min diff. = ?

$\frac{dD}{dx} = 3x^2 - 6x$

$0 = 3x(x-2)$

$x = 0$ $\boxed{x = 2}$

Not a min



7.) Max volume = ?

$V = (12-2x)(12-2x)x$

$= (144 - 48x + 4x^2)x$

$= 144x - 48x^2 + 4x^3$

$\frac{dV}{dx} = 144 - 96x + 12x^2$

$= 12(x^2 - 8x + 12) = 12(x-6)(x-2)$

$x=6$ $x=2$

$x = 2$ $12-2(2)$

$\boxed{2'' \times 8'' \times 8''}$